MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 7

SPRING 2019

DR. ABDUL-RAHMAN

(1) Show that if the random variables X and Y take only two values 0 and 1 and $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y)$, then X and Y are independent.

(2) Show that $\mathbb{E}\big((X-\alpha)^2\big)$ is minimized when $\alpha = \mathbb{E}(X)$.

(3) Is $M(t) = \frac{3}{10}e^t + \frac{1}{5}e^{-t} + \frac{1}{2}$ a moment generating function of a random variable? If yes, find the corresponding probability mass function. Then find the n-th moment of that random variable.

(4) Suppose that for a random variable X, we have $\mathbb{E}(X^n) = 5^n$ for all $n = 1, 2, \ldots$ Calculate the moment generating function of X.

(5) Let $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ be two independent random variables. Find the moment generating function of $Z = X + Y$. What is the distribution of Z.
Remark: If the sum of two independent random variables of the same type is a random variable that same type, we call that type *stable*.

(6) Let $X \sim \text{binomial}(n_1, p_1)$ and $Y \sim \text{binomial}(n_2, p_2)$ be two independent random variables. Find the moment generating function of $Z = X + Y + \mu$, where $\mu \in \mathbb{C}$.

(7) Show that for any random variable X, and constants $a, b \in \mathbb{C}$

$$\mathbb{M}_{aX+b}(t) = e^{bt}\mathbb{M}_X(at), \text{ for all } t \in \mathbb{R}.$$

(8) If X is a discrete uniform random variable on $\{0, 1, \ldots, a\}$, i.e.,

$$\mathbb{P}(X = k) = \frac{1}{1+a}, \text{ for } k = 0, 1, 2, \ldots, a,$$

show that X has moment generating function

$$\mathbb{M}_X(t) = \frac{1 - e^{t(a+1)}}{(1+a)(1-e^t)}, \ t \in \mathbb{R}$$